

Validation of min-max and max-min modified discrete uniform distributions as reliability models using pseudorandom number generators in Python

Valentina ASTAFI, Alexei LEAHU

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Abstract

This paper presents a statistical validation study through Monte Carlo simulation of the Min-Max and Max-Min modified discrete uniform distributions, proposed as a new dynamic lifetime distribution for serial-parallel and parallel-serial networks with independent, identically, discrete uniformly distributed component lifetimes. System lifetimes are simulated using five modern pseudorandom number generators: MT19937, PCG64, PCG64DXSM, Philox, and SFC64, each evaluated with ten distinct seed values and a minimum of five million simulations per run. The simulated statistics including mean, variance, skewness and kurtosis coefficients, entropy, lag-1 autocorrelation, the Kolmogorov-Smirnov test, and the Chi-square test are compared against exact theoretical values. Beyond the validation itself, the results confirm the high accuracy of all five generators in reproducing the theoretical distributions, with SFC64 achieving the best overall score in the proposed multi-criteria ranking.

Keywords: *lifetime distribution; max-min distribution; min-max distribution; network reliability; parallel-serial networks; serial-parallel networks*

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