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Mathematical models and system-level analysis of modern geolocation technologies

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Abstract

Accurate geolocation is essential for modern cyber-physical systems supporting applications such as autonomous navigation, intelligent transportation, mobile communications, and Internet of Things infrastructures. Contemporary solutions rely on complementary technologies, including Global Navigation Satellite Systems (GNSS), terrestrial wireless networks, and inertial sensors. This paper presents a theoretical system-level analysis of major geolocation technologies and their mathematical models, examining GNSS trilateration with clock bias, multi-constellation architectures, RSSI-based cellular and Wi-Fi localization, and inertial navigation. Estimation frameworks, including weighted least-squares and Kalman filtering for multi-sensor fusion, are also analyzed. A comparative evaluation of satellite, terrestrial, and inertial approaches highlights their operational characteristics, error sources, and accuracy regimes, emphasizing the importance of hybrid architectures for reliable positioning across diverse environments.

Keywords: *geolocation; kalman filtering; multi-sensor fusion; wi-fi localization*

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