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Analytical determination of fractional orders in FOPID controllers based on closed-loop dynamics

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Abstract

The paper presents an analytical method for determining the fractional orders of the integral and derivative components of the fractional PID controller. The approach is based on the analysis of the dominant pole pair dynamics of the closed loop system, characterized by the natural frequency and the damping ratio, as well as on the influence of these parameters on the fractional orders of the integral and derivative components of the controller. Based on this analysis, explicit analytical relations are proposed, in which the fractional orders are expressed as functions of the normalized deviations of the natural frequency and damping ratio from reference values, weighted by sensitivity coefficients that quantify the influence of the dynamic parameters of the system. A synthesis algorithm is presented in which, starting from an integer-order PID controller tuned by a suitable tuning method, the dynamic parameters of the closed loop system are identified and used to compute the fractional orders of the integral and derivative components. The proposed methodology reduces the dependence on numerical optimization procedures or heuristic tuning and provides a transparent and reproducible framework for the design of fractional controllers according to the desired closed loop dynamics. The effectiveness of the proposed method is demonstrated through two case studies.

Keywords: *analytical tuning; damping ratio; natural frequency*

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