The left and the right products, and the relative torsion theories

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$C_2\mathcal{V}$ is the topological vector locally convex spaces Hausdorff .

Definition 1. *(see [1]) Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory of category C. The pair $(\mathcal{K},\mathcal{R})$ is called a relative torsion theory (TTR), that is, relative to the subcategory $\mathcal{K}\cap\mathcal{R}$, if the functors $k : C \to \mathcal{K}$ and $r : C \to \mathcal{R}$ check the following two relationships:*

(1) The functors k and r commuted: $k \cdot r = r \cdot k$;

(2) For any object X of category C the square $r^X \cdot k^X = k^{rX} \cdot r^{kX}$ is pull-back and pushout.

Remark 1. *In the abelian categories a torsion theory $(\mathcal{T},\mathcal{F})$ is a TTR in relation to the intersection $\mathcal{T}\cap\mathcal{F} = 0$ [1].*

Let $\mathcal{K}$ (respectively $\mathcal{R}$) be o coreflective subcategory (respectively reflective) and the functors $k : C_2\mathcal{V} \to \mathcal{K}$ and $r : C_2\mathcal{V} \to \mathcal{R}$.

We note: $\mu\mathcal{K} = \{m \in \mathcal{M}ono|\ k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi|\ r(e) \in \mathcal{I}so\}$.

We examine the following conditions:

(S) The subcategory $\mathcal{K}$ is closed in relation to $\varepsilon\mathcal{R}$-factorobjects;

(D) The subcategory $\mathcal{R}$ is closed in relation to $\mu\mathcal{K}$-subobjects;

$\mathcal{K} *_s \mathcal{R}$ (respectively $\mathcal{K} *_d \mathcal{R}$) the left(respectively the right) product of subcategories $\mathcal{K}$ and $\mathcal{R}$ (see [3]).

Let $\widetilde{M}$ (respectively $\mathcal{S}$) be the coreflective subcategory of spaces with Mackey (respectively with weak locally convex) topology. Referring to the structure of factorization $(\mathcal{P}''(\mathbb{R}),\mathcal{I}''(\mathbb{R}))$ and $(\mathcal{E}'(\mathcal{K}),\mathcal{M}'(\mathcal{K}))$ see [2].

Theorem 1. *Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ - a nonzero reflective category of category $C_2\mathcal{V}$. The following stated are equivalent:*

(1) The pair $(\mathcal{K},\mathcal{R})$ forms a TTR.

(2) *(a) The coreflectork* $: C_2\mathcal{V} \longrightarrow \mathcal{K}$ *and reflector* $r : C_2\mathcal{V} \longrightarrow \mathcal{R}$ *functors commuted:* $kr = rk$*;*

(b) $\mathcal{K} *_s \mathcal{R} = \mathcal{K}$*;*

(c) $\mathcal{K} *_d \mathcal{R} = \mathcal{R}$*.*

(3) *(a) The functors k and r commuted:* $kr = rk$*;*

(b) The subcategory $\mathcal{K}$ *posed the property (S);*

(c) The subcategory $\mathcal{R}$ *posed the property (D).*
If $\mathcal{M} \subset \mathcal{K}$ *and* $\mathcal{S} \subset \mathcal{R}$ *then the previous conditions are equivalent to the following:*

(4) *(a) The functors k and r commuted:* $kr = rk$*;*

(b) The subcategory $\mathcal{K}$ *is* $\mathcal{I}''(\mathcal{R})$*-coreflective;*

(c) The subcategory $\mathcal{R}$ *is* $\mathcal{E}'(\mathcal{K})$*-reflective.*

References

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[3] A. Ţurcanu, *The factorization of reflector functors,* Buletinul Institutului Politehnic din Iasi, Romania, 2007, t. LIII (LVII), f.5, p. 377-391.

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