Computer Science Journal of Moldova, vol.7, no.2(20), 1999

Approximation procedures for data acquisition in testing systems

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Abstract

This paper deals with the results of spline approximation functions applied to periodical experimental data and its comparison with polynomial approximation functions. The experimental data processing methods based on spline functions ensure better quality of approximation, but the algorithms are very complicated. An algorithm that allows to exclude this disadvantage is presented in the paper. The main idea of the proposed algorithm is the calculation of spline function derivatives reduced to the algebraic linear equation system, which allows successful application in the testing and diagnosis systems.

1 Introduction

Testing and diagnosis of automobile systems, including the electrical and electronic equipment, is carried out through a large number of measurements of parameters with posterior processing. The measurements of automobile systems parameters are performed in extreme conditions of electro-magnetic disturbance. These lead to appearance of high level of chaotic noise in the acquisition data in testing systems. Application of schematic design methods can only reduce, but not exclude these random errors of data, therefore, it is necessary to process preliminary the acquired data as regards to filtration of the noise and their approximation. This preliminary processing of the acquired data, obtained at the testing of complex technical objects, in particular case, the automobile, must be done through approximation, interpolation, differentiation, and integration of the functions that reflect the experimental data. To solve this problem classical mathematical methods are used. These methods cannot obtain satisfactory results at the interdependent processing of the complex technical object parameters. For instance, derivatives of digital differentiation, obtained with classical mathematical methods on the experimental data that contain random errors differs very much from actual derivatives [3, 4, 5].

Lately it is observed an intensive application of a new branch of a modern mathematics – spline function theory, that permits to solve more effectively the problems of experimental data processing, acquired from technical objects with complex internal structure [2, 6, 7]. The analysis of results of experimental data processing methods [6, 7, 10] and its comparison with specific of the problems permits to select two experimental data approximation modalities: a polynomial approximation function and cubic spline approximation periodical function, because practically most of parameter's measurement processes are on the short interval of time. For instance, signals of the camshaft and crankshaft position sensors, signals of primary and secondary ignition circuit's voltage; signals of generator voltage and another signals are periodical processes. As criteria for selecting of these approximation modalities, adaptive elimination of noise and random errors and time performance is used, taking into consideration real time applications.

This paper deals with results of spline approximation periodical functions applied to experimental data and its comparison with polynomial approximation functions.

2 Algorithm 1: Experimental data processing with a polynomial approximation function

The main idea of this algorithm is value approximation of current point, taking into consideration the values of neighboring points with their inherent weight coefficients. Additionally, this algorithm has two important features:

a) experimental data are measured with strictly synchronizing signal of other process, for instance, crankshaft position;

b) degree of approximation can be selected to obtain a compromise solution between quality of approximation and calculation time.

Mathematical model of the algorithm:

a) The acquired data are presented in the following way:

$$R = \{S_j, < Y_{ij} >\}, \quad i = 1 \dots n, j = 1 \dots m, \tag{1}$$

where S_j – is the signal synchronized with the engine cylinders;

$< Y_{ij} >$ – the values of acquired data of the measured signal at the each rotation of the crankshaft;

n, m– respectively the number of acquired data at the one crankshaft rotation and the number of engine cylinders.

b) The approximation function has the follow view:

$$\Phi_i = \frac{(k-1) \cdot Y_i + \sum_{j=1}^{k-2} (k-(j+1)) \cdot Y_{i-(k-(j+1))}}{(k+1)} + \frac{\sum_{j=1}^{k-2} (k-(j+1)) \cdot Y_{i+(k-(j+1))}}{(k+1)} \tag{2}$$

where k - the number of values, including the current value, on base which the approximation is performed.

Step 1. Data acquisition is performed:

$$R = \{S_j, < Y_{ij} >\}, \quad i = 1 \dots n, j = 1 \dots k.$$

Step 2. Set $k = 3$ or $5, 7, 9 \dots$.

Step 3. Set $i = k - 1$.

Step 4. Calculate the value of antecedent neighboring points with their inherent weight coefficients:

$$S_a = (k-(j+1)) \cdot Y_{i-(k-(j+1))}, \quad \text{for} \quad j = 1 \quad \text{to} \quad k-2.$$

Step 5. Calculate the value of succedent neighboring points with their inherent weight coefficients:

$$S_s = (k - (j+1)) \cdot Y_{i+(k-(j+1))}, \quad \text{for} \quad j = 1 \quad \text{to} \quad k-2.$$

Step 6. Calculate the value of current point

$$\Phi_i = \frac{((k-1) \cdot Y_i + S_a + S_s}{(k+1)}.$$

Step 7. Set $i = i + 1$. If $i < n - (k-1)$, go to **S**tep 4. Otherwise stop.

The experimental data approximation values are $\Phi_i = F(Y_i)$.

3 Algorithm 2: Experimental data processing with a spline approximation function

Application of a spline function theory allows more effective solution of the problems of experimental data processing, acquisitioned from technical objects with complex internal structure [2, 6, 7]. The experimental data processing methods based on spline functions ensure better quality of approximation [2, 6, 7, 10], but the algorithms are very complicated. In this paper the algorithm is proposed that allows excluding the imperfection of above-named algorithms. The important features of the proposed algorithm are the following:

a) experimental data must be measured with strictly synchronizing signal of other process, for correctness segmentation of these data;

b) calculation of spline function derivatives is reduced to the solution of algebraic linear equation system;

c) degree of approximation can be selected to obtain a compromise between quality of approximation and calculation time.

Mathematical model of the algorithm:

a) the acquired data are presented in the following way:

$$R = \{S_j, < X_i, Y_{ij} >\}, \quad i = 1 \ldots n, j = 1 \ldots m, \tag{3}$$

where S_j – is the signal synchronized with the engine cylinders;

$< X_i, Y_{ij} >$ – the values of acquired data and its coordinates of the measured signal at the each rotation of the crankshaft;

n, m – respectively the number of acquired data at the one crankshaft rotation and the number of engine cylinders.

b) the approximation cubic spline function has the following view:

$$\begin{aligned} \varphi(x_j) &= K_{j-1}\frac{(x_j - x)^3}{6h_j} + K_j\frac{(x - x_{j-1})^3}{6h_j} + \\ &+ \frac{(x_j - x)}{h_j}(y_{j-1} - \frac{K_{j-1}h_j^2}{6}) + \ldots \\ &\ldots + \frac{(x - x_{j-1})}{h_j}(y_j - \frac{K_j h_j^2}{6}), \end{aligned} \tag{4}$$

where $K_j = \varphi''(x_j); h_j = x_j - x_{j-1}; j = 2 \ldots N$.

Definition 1 *The cubic spline function is named a function $y = \varphi(x)$, that is twice differentiated on the interval $h_j = x_j - x_{j-1}$, and that coincided with cubic polynomial and satisfied interpolation condition $\varphi(x_j) = y_j$, for $j = 1 \ldots N$.*

Note 1 *It is decided to use cubic spline functions, because these functions allow to obtain the best quality of approximation [7, 10].*

c) calculation of cubic spline functions usually is performed by minimization of the following functional [7, 10]:

$$\Psi(K_1, K_2, \ldots, K_N, z_1, z_2, \ldots, z_N, \lambda) =$$
$$\sum_{j=2}^{N} \int_{x_{j-1}}^{x_j} [K_{j-1}\frac{(x_j - x)}{h_j} + K_j\frac{(x - x_{j-1})}{h_j}]^2 dx + \ldots$$

$$\ldots + \lambda\{\sum[K_{j-1}\frac{(x_j - x)^3}{6h_j} + K_j\frac{(x - x_{j-1})^3}{6h_j} +$$
$$+\frac{(x_j - x)}{h_j}(z_{j-1} - \frac{K_{j-1}h_j^2}{6}) + \ldots \quad (5)$$
$$\ldots + \frac{(x - x_{j-1})}{h_j}(y_j - \frac{K_j h_j^2}{6} - y_j)]^2 - CN\delta^2\},$$

where $C > 0 (C \cong 2 \div 3)$ – parameter that characterizes the level of authenticity.

Global minimum of this function can be obtained by diverse optimization methods (for instance, by gradient method), that require long run time. Despite the high accuracy of these methods, they cannot be applicable for testing of complex technical objects, that require about $10 \div 16$ thousands of experimental data measurements in real time. To find the solution of this problem it is proposed to minimize the following function:

$$\Psi(\varphi) = \int_a^b [\varphi''(x)^2]dx + \sum_{j=1}^{N} p_j^{-1}[\varphi(x_j) - y_j]^2, \quad (6)$$

where $\varphi(x_j)$ – spline function's value in the coordinate x_j;
p_j – weight coefficients.

It is also proposed to reduce the calculation of spline function derivatives K_j to the solution of the following algebraic linear equation system:

$$\begin{aligned}
d_1 &= c_1K_1 + b_1K_2 + a_3K_3 \\
d_2 &= b_1 + c_2K_2 + b_2K_3 + a_2K_4 \\
& \ldots\ldots\ldots\ldots\ldots \\
d_i &= a_iK_{i-2} + b_{i-1}K_{i-1} + c_iK_i + b_iK_{i+1} + a_iK_{i+2}, \quad (7) \\
& i = 3, \ldots, N \\
& \ldots\ldots\ldots\ldots\ldots \\
d_{N-1} &= a_{N-3}K_{N-3} + b_{N-2}K_{N-2} + c_{N-1}K_{N-1} + b_{N-1}K_{N-1} \\
d_N &= a_{N-2}K_{N-2} + b_{N-1}K_{N-1} + c_NK_N.
\end{aligned}$$

The coefficients of the above algebraic linear equation system are determined as follows:

$$
\begin{aligned}
a_i &= \frac{p_{i+1}}{h_{i+1}h_{i+2}};\\
b_i &= \frac{h_{i+1}}{6} - \frac{1}{h_{i+1}}\left[\left(\frac{1}{h_i}+\frac{1}{h_{i+1}}\right)p_i + \left(\frac{1}{h_{i+1}}+\frac{1}{h_{h+2}}\right)p_{i+1}\right] \qquad (8)\\
c_i &= \frac{h_i+h_{i+1}}{3} + \frac{1}{h_i^2}p_{i-1} + \left(\frac{1}{h_i}+\frac{1}{h_{i+1}}\right)^2 p_i + \frac{1}{h_{i+1}^2}p_{i+1};\\
d_i &= \left(\frac{y_{i+1}-y_i}{h_{i+1}} - \frac{y_i-y_{i-1}}{h_i}\right) \text{for} i = 2,\ldots,N. \qquad (9)
\end{aligned}
$$

d) After calculation of derivatives K_j the values of spline function are determined:

$$z_i = y_i - p_i D_i, i = 1,\ldots,N, \qquad (10)$$

where coefficients $D_1,\ldots,D_N$ and D_i will be calculated using the following relations for $i = 2,\ldots,N-1$:

$$
\begin{aligned}
D_1 &= \frac{K_2-K_1}{h_2}; D_N = -\frac{K_N-K_{N-1}}{h_N};\\
D_i &= -\frac{K_{i-1}-K_i}{h_{i+1}} - \frac{K_i-K_{i-1}}{h_i}. \qquad (11)
\end{aligned}
$$

Step 1. Data acquisition is performed:

$$R = \{S_j, < X_i, Y_{ij} >\}, i = 1,\ldots n, j = 1,\ldots,k.$$

Step 2. First subset of data is selected

$$R_1 = \{S_1, < X_i, Y_{i1} >\}, i = 1,\ldots,n.$$

Step 3. Set $p_i = 0.005$ or $0.001\ldots0.5$ for $i = 1$ to n.

Step 4. Set $h_1 = x_2 - x_1; h_2 = x_3 - x_2$.

Step 5. Calculate coefficient a_1 of the algebraic linear equation system:

$$a_1 = \frac{p_2}{h_1 \cdot h_2}.$$

Step 6. Calculate coefficient b_1 of the algebraic linear equation system:

$$b_1 = \frac{h_1}{6} - \frac{1}{h_1}[(\frac{1}{h_1} + \frac{1}{h_2})p_1 + (\frac{1}{h_1} + \frac{1}{h_2})p_2].$$

Step 7. Calculate coefficient c_1 of the algebraic linear equation system:

$$c_1 = \frac{h_1 + h_2}{3} + \frac{1}{h_i^2}p_1 + (\frac{1}{h_1} + \frac{1}{h_2})^2 p_1 + \frac{1}{h_2^2}p_2.$$

Step 8. Calculate coefficient d_1 of the algebraic linear equation system:

$$d_1 = (\frac{y_2 - y_1}{h_2} - \frac{y_1}{h_1}).$$

Step 9. Calculate $h_{N-1} = x_N - x_{N-1}; h_{N-2} = x_{N-1} - x_{N-2i}$.

Step 10. Calculate coefficient a_N of the algebraic linear equation system:

$$a_N = \frac{p_N}{h_{N-2} \cdot h_{N-1}}.$$

Step 11. Calculate coefficient b_N of the algebraic linear equation system:

$$b_N = \frac{h_{N-2}}{6} - \frac{1}{h_{N-2}}[(\frac{1}{h_{N-2}} + \frac{1}{h_{N-1}})p_1 + (\frac{1}{h_{N-2}} + \frac{1}{h_{N-1}})p_2].$$

Step 12. Calculate coefficient c_N of the algebraic linear equation system:

$$c_N = \frac{h_{N-2} + h_{N-1}}{3} + \frac{1}{h_{N-2}^2}p_1 + (\frac{1}{h_{N-2}} + \frac{1}{h_{N-1}})^2 p_1 + \frac{1}{h_{N-1}^2}p_2.$$

Step 13. Calculate coefficient d_N of the algebraic linear equation system:

$$d_N = (\frac{y_N}{h_{N-1}} - \frac{y_N - y_{N-1}}{h_{N-2}}).$$

Step 14. Set $i = 2$.

Step 15. Calculate $h_{i+1} = x_{i+2} - x_{i+1}; h_i = x_{i+1} - x_i$.

Step 16. Calculate coefficient a_i of the algebraic linear equation system:

$$a_i = \frac{p_{i+1}}{h_{i+1} \cdot h_{i+2}}.$$

Step 17. Calculate coefficient b_i of the algebraic linear equation system:

$$b_i = \frac{h_{i+1}}{6} - \frac{1}{h_{i+1}}[(\frac{1}{h_i} + \frac{1}{h_{i+1}})p_i + (\frac{1}{h_{i+1}} + \frac{1}{h_{h+2}})p_{i+1}].$$

Step 18. Calculate coefficient c_i of the algebraic linear equation system:

$$c_i = \frac{h_i + h_{i+1}}{3} + \frac{1}{h_i^2}p_{i-1} + (\frac{1}{h_i} + \frac{1}{h_{i+1}})^2 p_i + \frac{1}{h_{i+1}^2}p_{i+1}.$$

Step 19. Calculate coefficient d_i of the algebraic linear equation system:

$$d_i = (\frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i+1}}{h_i}).$$

Step 20. Set $i = i + 1$. If $i < n - 1$, go to **S**tep 15.

Step 21. Set initial values: $E_1, F_1, E_{N-1}, E_N, D_N = 0$.

Step 22. Calculate coefficients:

$$R_1 = -\frac{d_1}{c_1}; Q_1 = -\frac{E_1}{c_1}.$$

Step 23. Calculate derivation:

$$K_1 = \frac{F_1}{c_1}.$$

Step 24. Calculate additional coefficient: $G = c_2 + b_2 \cdot R_1$.

Step 25. Set $i = 2$.

Step 26. Calculate coefficient: $W = a_i \cdot R_{i-2} + b_i$.

Step 27. Calculate coefficient: $G = c_i + W \cdot R_{i-1} + a_i \cdot Q_{i-2}$.

Step 28. Calculate coefficients:

$$R_i = -\frac{c_i + W \cdot Q_{i-1}}{G}.$$

Step 29. Calculate coefficients:

$$Q_i = -\frac{p_{i-1}}{G \cdot h_i \cdot h_{i-1}}.$$

Step 30. Calculate derivation:

$$K_i = \frac{R_i - WK_{i-1} - a_i K_{i-2}}{G}.$$

Step 31. Set $i = i + 1$. If $i < n$, go to **S**tep 25.

Step 32. Calculate derivation:

$$K_{N-1} = \frac{R_N K_{i-1} + K_{i-2}}{G}.$$

Step 33. Set $i = 2$.

Step 34. Recalculate derivation: $K_1 = R_i K_{i-1} + Q_i K_{i-2} + K_1$.

Step 35. Set $i = i + 1$. If $i < n$, go to **S**tep 33.

Step 36. Set $j = 1$.

Step 37. Calculate the value of first point:

$$D_1 = \frac{K_2 - K_1}{x_2 - x_1}; z_{11} = y_{11} - p_1 D_1.$$

Step 38. Calculate the value of last point:

$$D_N = \frac{K_N - K_{N-1}}{x_N - x_{N-1}}; z_{N1} = y_{N1} - p_N D_N.$$

Step 39. Set $i = 2$.

Step 40. Calculate $h_{i+1} = x_{i+2} - x_{i+1}; h_i = x_{i+1} - x_i$.

Step 41. Calculate coefficient:

$$D_i = \frac{K_{i+1} - K_i}{h_{i+1}} - \frac{K_i - K_{i-1}}{h_i}.$$

Step 42. Set $j = 1$.

Step 43. Calculate the value of spline function in x_i point: $z_{ij} = y_{ij} - p_i D_i$.

Step 44. Set $j = j + 1$. If $j < m$, go to **S**tep 42.

Step 45. Set $i = i + 1$. If $i < n - 1$, go to **S**tep 39. Otherwise stop.

4 Testing and complexity of algorithms

The proposed algorithm was tested from the point of view of influence of weight coefficients (k – for algorithm 1, p - for algorithm) on the quality of approximations. The quality of approximation is evaluated by the following relation:

$$|\Phi_i - Y_i| < \delta, i = 1, \dots, n, \tag{12}$$

where δ – error values of measurements.

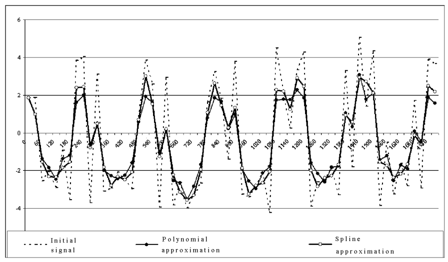

Figure 1. Approximation rezults of testing function
$y = 7\sin(4x) + 5\cos(8x)$.

The testing of algorithms was performed with diverse conditions and results of both algorithms approximation were compared. For instance, some results with conditions of testing are the following:

– testing function $y = 7\sin(4x)+5\cos(8x)$ values, disturbed by 15% noise/signal ratio approximation with polynomial function (Algorithm 1, $k = 5$) and spline function (Algorithm 2, $p = 0.05$) (Fig. 1);

– mass air flow sensor signal approximation with polynomial function (Algorithm 1, $k = 5$) and spline function (Algorithm 2, $p = 0.05$) (Fig. 2).

Analyzing results of testing of algorithms and taking into consideration, that, usually, δ – error value of measurement is in the limits of $2.5\% < \delta < 10\%$, one can make the following conclusions:

– approximation quality of polynomial function (Algorithm 1) with $k = 3, 5, 7$ satisfy the condition (12) in the most of cases (see Fig. 1 and Fig. 2);

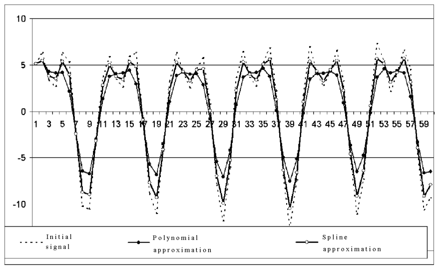

Figure 2. Approximation rezults of mass air flow sensor signal.

– approximation quality of cubic spline function (Algorithm 2) with $p = 0.001 \div 0.5$ also satisfy the condition (12) in the most of cases (see Fig. 1 and Fig. 2).

The polynomial that characterizes the computational efficiency of the algorithm was determined according to known methods [1]. The factors that influence the most the efficiency of these algorithms are the number of experimental data and number of neighboring points taken into consideration. The polynomials of the algorithms 1 and 2 are of the first degree:

$$O(N) \cong C_1 \cdot N_{mas} + C_2 \cdot K_s + C_3 \cdot K_s + C_0 \qquad (13)$$

$$O(N) \cong C_1 \cdot N_{mas} + C_2 \cdot N_{mas} + C_3 \cdot N_{mas} + C_4 \cdot N_{mas} + C_0 \qquad (14)$$

These algorithms depend linearly from the number of experimental data, that's why the efficiency of the algorithm remains high. The simulation and algorithm's realization results confirm its computational efficiency. For instance, the Algorithm 1 for 3 neighboring points was implemented with 16 IBM PC instructions, that are running in less than $10\mu s$ [8, 9]; the Algorithm 2 was implemented in C++ and runs in $5 \div 6$ sec for 10 thousands of experimental data [8, 9]. Therefore, the approximation Algorithm 2 is recommended to be used for such cases as: testing of cylinder's compression, testing of generator pulsation, exhaust gas analysis and other procedures.

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Received 28 July, 1999