LANCED MULTI-PORT ELECTRIC NETWORK AND ITS PROJECTIVE COORDINATES

A. Penin and A. Sidorenko

Gitsu Institute of Electronic Engineering and Nanotechnologies,
Academy of Sciences of Moldova
E-mails: aapenin@mail.ru; anatoli.sidorenko@kit.edu

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Abstract

For an active multi-port network of direct current, as a model of distributed power supply system, the problem of recalculation of the changeable load currents is considered. Conditions of utilization of projective coordinates for the interpretation of changes or "kinematics" of regime parameters of the network are determined. Therefore, changes in regime parameters are introduced by the cross ratio of four points. Easy-to-use formulas of the recalculation of the currents, which possess the group properties at a change in the conductivity of the loads, are obtained to express the final values of currents through the intermediate changes in the load currents and conductivities. The obtained results contribute to the development of the basics of the electric circuit theory.

1. Introduction

In the electric circuit theory, attention is given to the networks with variable parameters of elements. In particular, a new method, which can determine the functional dependence of any circuit variable with respect to any set of design variables, is presented in [1].

At present, special consideration is given to distributed renewable power supply systems with a lot of loads and voltage sources [2–5]. In turn, a particular problem of convenient recalculation of changeable load currents is raised. The conventional approach uses the changes in load conductivities in the form of increments. Recalculation of currents leads to the solution of a system of algebraic equations of a corresponding order. Therefore, for a number or group of changes in these conductivities, these increments should be counted concerning an initial circuit and the solution of the equations system is repeated. So, this nonfulfilment of group properties (when the final result should be obtained through intermediate results) complicates recalculation and limits the capabilities of this approach.

An approach for the interpretation of changes or "kinematics" of the circuit regimes on the basis of projective geometry is represented in [6]. The changes in regime parameters are introduced otherwise. Therefore, as if obvious changes in the form of increments are formal and do not reflect the substantial aspect of the mutual influences: conductivities → currents. The offered approach allows obtaining the convenient formulas of recalculation of load currents. In particular, a network with a common node for a lot of loads is also shown [7, 8]. In this context, it is important to consider the general structure of the network.

2. Projective coordinates of an active two-port network

Consider an active two-port with changeable load conductivities Y_{L1}, Y_{L2} in Fig. 1a.

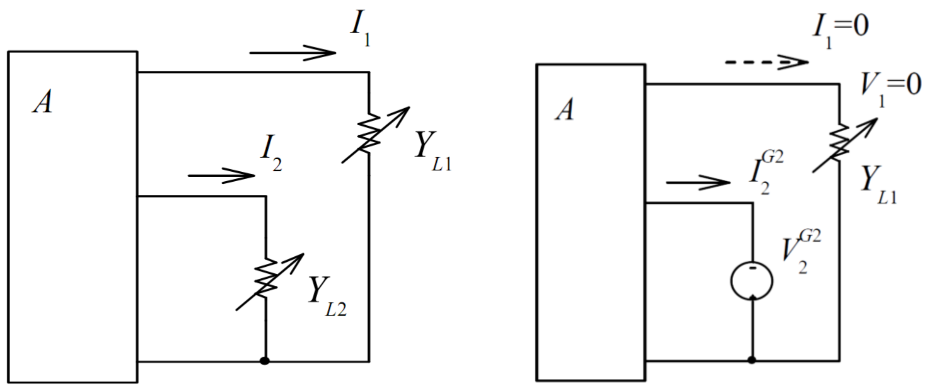

Fig. 1. Active two-port (**a**) and active two-port regime with the second load base parameters (**b**).

Let us give the necessary relationships for this circuit [7]. The circuit is described by the following system of the Y - parameters equations

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} \\ Y_{12} & -Y_{22} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \end{pmatrix}, \tag{1}$$

where I_1^{SC}, I_2^{SC} are the short circuit SC currents.

Taking into account the voltages $V_1 = I_1 / Y_{L1}$, $V_2 = I_2 / Y_{L2}$, two bunches of load straight lines with parameters Y_{L1}, Y_{L2} are shown in Fig. 2.

The bunch center, point G_2, corresponds to the bunch with parameter Y_{L1}. The bunch center corresponds to such a regime of the load Y_{L1} that does not depend on its values. It is carried out for $I_1 = 0, V_1 = 0$ at the expense of the characteristic regime parameter of the second load in Fig. 1b:

$$I_2 = I_2^{G2}, \quad V_2 = V_2^{G2}, \quad Y_{L2} = \frac{I_2}{V_2} = Y_{L2}^{G2} . \tag{2}$$

The parameters of the center G_1 of bunch Y_{L2} have the similar form. Another form of the characteristic regime is the short circuit regime of both loads ($Y_{L1} = \infty$, $Y_{L2} = \infty$) that is presented by point SC. The open circuit regime of both loads is also characteristic and corresponds to the origin of coordinates, point 0.

Let the initial or running regime correspond to point M^1, which is set by the values of conductivities Y_{L1}^1, Y_{L2}^1 or currents I_1^1, I_2^1 of the loads. Also, this point is defined by the projective non-uniform m_1^1, m_2^1 and homogeneous $\xi_1^1, \xi_2^1, \xi_3^1$ coordinates which are set by a reference triangle $G_1 \; 0 \; G_2$ and a unit point SC [7, 9]. Point 0 is the origin of coordinates and straight line $G_1 G_2$ is the line of infinity ∞.

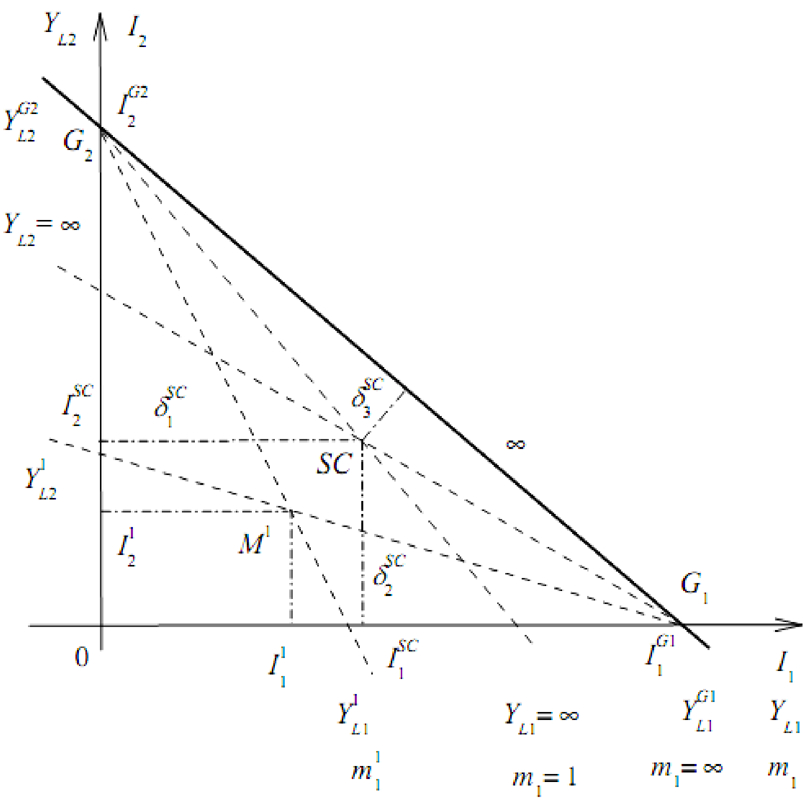

Fig. 2. Two bunches of load straight lines with the parameters Y_{L1}, Y_{L2}.

The non-uniform projective coordinate m_1^1 is set by a cross ratio of four points, three of them correspond to the points of the characteristic regimes, and the fourth corresponds to the point of the running regime

$$m_1^1 = (0\ Y_{L1}^1\ \infty\ Y_{L1}^{G1}) = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \div \frac{\infty - 0}{\infty - Y_{L1}^{G1}} = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}}. \tag{3}$$

Here, the points $Y_{L1} = 0$, $Y_{L1} = Y_{L1}^{G1}$ correspond to the extreme or base values. The point of $Y_{L1} = \infty$ is a unit point. These values of m_1 are also shown in Fig. 2. For the point $Y_{L1}^1 = Y_{L1}^{G1}$, the projective coordinate $m_1 = \infty$ defines the sense of line of infinity $G_1 G_2$. The cross ratio for the projective coordinate m_2^1 is expressed similarly. The homogeneous projective coordinates ξ_1, ξ_2, ξ_3 set the non-uniform coordinates as follows:

$$m_1 = \frac{\xi_1}{\xi_3} = \frac{\rho\xi_1}{\rho\xi_3}, \quad m_2 = \frac{\xi_2}{\xi_3} = \frac{\rho\xi_2}{\rho\xi_3}, \tag{4}$$

where ρ is a coefficient of proportionality.

The homogeneous coordinates are defined by the ratio of the distances of the points M^1, SC to the sides of the reference triangle:

$$\rho\,\xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}} = \frac{I_1^1}{I_1^{SC}}, \quad \rho\,\xi_2^1 = \frac{I_2^1}{I_2^{SC}}, \quad \rho\,\xi_3^1 = \frac{\delta_3^1}{\delta_3^{SC}}.$$

For finding distances $\delta_3^1, \delta_3^{SC}$ to straight line $G_1 G_2$, the equation of this straight line is used. Then

$$\left(\frac{I_1^1}{I_1^{G1}}+\frac{I_2^1}{I_2^{G2}}-1\right)=\mu_3\delta_3^1, \qquad \left(\frac{I_1^{SC}}{I_1^{G1}}+\frac{I_2^{SC}}{I_2^{G2}}-1\right)=\mu_3\delta_3^{SC},$$

where μ_3 is a normalizing factor.

The homogeneous coordinates have a matrix form

$$\rho[\xi^1]=[\mathbf{C}]\cdot[\mathbf{I}^1], \tag{5}$$

where matrix and vectors

$$[\mathbf{C}]=\begin{pmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G2}\mu_3\delta_3^{SC}} & -\frac{1}{\mu_3\delta_3^{SC}} \end{pmatrix}, \quad [\mathbf{I}^1]=\begin{pmatrix} I_1^1 \\ I_2^1 \\ 1 \end{pmatrix}, \quad [\xi^1]=\begin{pmatrix} \xi_1^1 \\ \xi_2^1 \\ \xi_3^1 \end{pmatrix}.$$

From here, we may pass to the non-uniform coordinates

$$m_1^1=\frac{\rho\xi_1^1}{\rho\xi_3^1}=\frac{I_1^1/I_1^{SC}}{\frac{I_1^1}{I_1^{G1}}+\frac{I_2^1}{I_2^{G2}}-1}\mu_3\delta_3^{SC}, \quad m_2^1=\frac{\rho\xi_2^1}{\rho\xi_3^1}=\frac{I_2^1/I_2^{SC}}{\frac{I_1^1}{I_1^{G1}}+\frac{I_2^1}{I_2^{G2}}-1}\mu_3\delta_3^{SC}. \tag{6}$$

The inverse transformation of (5)

$$\rho[\mathbf{I}^1]=[\mathbf{C}]^{-1}\cdot[\xi^1], \quad [\mathbf{C}]^{-1}=\begin{pmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G1}} & \frac{I_2^{SC}}{I_2^{G2}} & -\mu_3\delta_3^{SC} \end{pmatrix}. \tag{7}$$

From here, we pass to the currents

$$I_1^1=\frac{\rho I_1^1}{\rho 1}=\frac{I_1^{SC}m_1^1}{\frac{I_1^{SC}}{I_1^{G1}}m_1^1+\frac{I_2^{SC}}{I_2^{G2}}m_2^1-\mu_3\delta_3^{SC}}=\frac{I_1^{SC}m_1^1}{\frac{I_1^{SC}}{I_1^{G1}}(m_1^1-1)+\frac{I_2^{SC}}{I_2^{G2}}(m_2^1-1)+1},$$

$$I_2^1=\frac{\rho I_2^1}{\rho 1}=\frac{I_1^{SC}m_2^1}{\frac{I_1^{SC}}{I_1^{G1}}(m_1^1-1)+\frac{I_2^{SC}}{I_2^{G2}}(m_2^1-1)+1}. \tag{8}$$

Obtained transformation allows finding currents I_1, I_2 for the preset values of conductivities Y_{L1}, Y_{L2} by using coordinates m_1, m_2. Furthermore, we note these expressions (6) and (8) have the common form, which is convenient for practice.

We consider now the recalculation of the load currents. Let a subsequent regime correspond to point M^2 with load parameters $Y_{L1}^2,\ Y_{L2}^2,\ I_1^2,\ I_2^2$.

The non-uniform $m_1^2,\ m_2^2$ coordinates are defined similarly to (3). Therefore, the regime changes m_1^{21}, m_2^{21} are naturally expressed through the cross ratio

$$m_1^{21} = (0\ Y_{L1}^2\ Y_{L1}^1\ Y_{L1}^{G1}) = m_1^2 \div m_1^1,\ m_2^{21} = m_2^2 \div m_2^1 .$$

We also define the homogeneous coordinates of the point M^2 and represent nonuniform coordinates m_1^2 and m_2^2 in the form

$$m_1^2 = m_1^{21}\frac{\xi_1^1}{\xi_3^1},\ m_2^2 = m_2^{21}\frac{\xi_2^1}{\xi_3^1}.$$

Using (7) and (8), we immediately obtain the required currents

$$I_1^2 = \frac{I_1^1 m_1^{21}}{\frac{I_1^1}{I_1^{G1}}(m_1^{21}-1)+\frac{I_2^1}{I_2^{G2}}(m_2^{21}-1)+1},\quad I_2^2 = \frac{I_2^1 m_2^{21}}{\frac{I_1^1}{I_1^{G1}}(m_1^{21}-1)+\frac{I_2^1}{I_2^{G2}}(m_2^{21}-1)+1}. \tag{9}$$

The obtained relationships carry out the recalculation of currents at a respective change in load conductivities. These relations are the projective transformations and possess group properties.

3. Active multi-port network with a common node for loads

Consider the active multi-port network in Fig. 3 with given elements and a common node N for load conductivities Y_{L1}, Y_{L2}, Y_{L3}. In particular, internal conductance y_{0N} of voltage source V_0 defines the mutual influence of the loads.

A circuit is described by the following system of the Y-parameters equation

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -Y_{11} & Y_{12} & Y_{13} \\ Y_{12} & -Y_{22} & Y_{23} \\ Y_{13} & Y_{23} & -Y_{33} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} + \begin{pmatrix} I_1^{SC} \\ I_2^{SC} \\ I_3^{SC} \end{pmatrix}, \tag{10}$$

where I_1^{SC}, I_2^{SC}, and I_3^{SC} are the short circuit SC currents of all the loads.

Similarly to the above, let us give the necessary relationships and geometrical interpretation for this active multi-port. We accept that coordinate axes I_1, I_2, I_3 determine the three-dimensional Cartesian coordinate system (I_1, I_2, I_3) in Fig. 4.

Taking into account voltages $V_1 = I_1 / Y_{L1}, V_2 = I_2 / Y_{L2}$, and $V_3 = I_2 / Y_{L3}$, the equations of three bunches of planes are obtained in the form

$$(I_1, I_2, I_3, Y_{L1}) = 0,\ (I_1, I_2, I_3, Y_{L2}) = 0,\ (I_1, I_2, I_3, Y_{L3}) = 0 .$$

Crossing of the planes of one bunch among themselves defines a bunch axis. The equation of the axis of bunch Y_{L1} corresponds to the condition $I_1 = 0, V_1 = 0$ and to equation $(I_2, I_3) = 0$. Therefore, this axis is located in the plane of I_2, I_3 in Fig. 4 and determines the points of

intersection or the base values of I_2^{G2}, I_3^{G3}. Similarly, we get the base value of I_1^{G1}.
Thus, we accept the $G_1G_2G_3$ plane, which passes through these three points I_1^{G1}, I_2^{G2}, and I_3^{G3}, as the plane of infinity ∞. So, we get the coordinate tetrahedron $0G_1G_2G_3$.

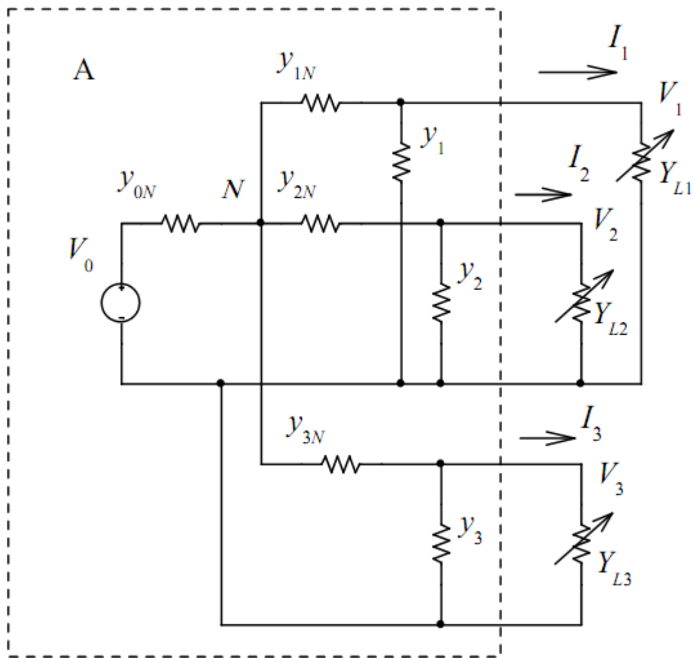

Fig. 3. Active multi-port with a common node N .

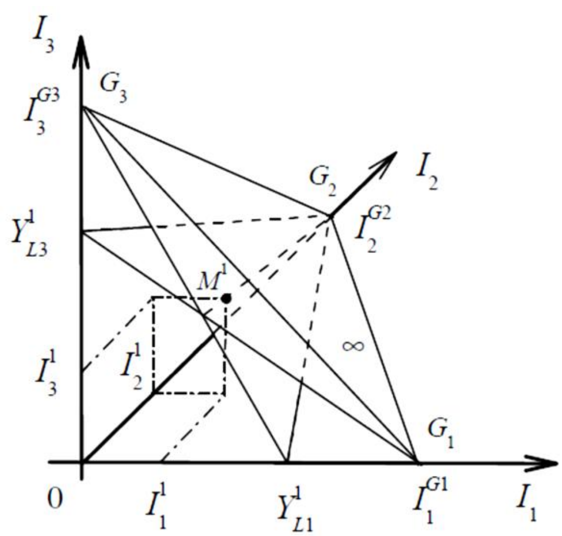

Fig. 4. Cartesian coordinate system (I_1, I_2, I_3) and projective coordinate $0G_1G_2G_3$.

Let the initial or running regime correspond to point M^1, which is set by load conductivities $Y_{L1}^1, Y_{L2}^1, Y_{L3}^1$ and currents I_1^1, I_2^1, I_3^1. Then, the running value Y_{L3}^1 corresponds to the plane which passes through point M^1 and straight line G_1G_2. This line corresponds to the intersection of the $G_1G_2G_3$, $0G_1G_2$ planes.

Similarly, the Y_{L2}^1 value agrees with the plane that passes through point M^1 and straight line G_1G_3.

In addition, the Y_{L1}^1 value matches point M^1 and straight line G_2G_3. We recall that this line is the axis of bunch plane Y_{L1}. In turn, the values of $Y_{L1}=0$, $Y_{L1}=\infty$, and Y_{L1}^{G1} are the characteristic values of Y_{L1}. Therefore, the running value Y_{L1}^1 corresponds to the non-uniform coordinate m_1^1 in the form of cross ratio of four points:

$$m_1^1=(0\ Y_{L1}^1\ \infty\ Y_{L1}^{G1})=\frac{Y_{L1}^1}{Y_{L1}^1-Y_{L1}^{G1}}\div\frac{\infty-0}{\infty-Y_{L1}^{G1}}=\frac{Y_{L1}^1}{Y_{L1}^1-Y_{L1}^{G1}}.$$

There, points $Y_{L1}=0$, $Y_{L1}=Y_{L1}^{G1}$ are the base points and $Y_{L1}=\infty$ is a unit point.

The cross ratio for m_2^1, m_3^1 is expressed similarly:

$$m_2^1=(0\ Y_{L2}^1\ \infty\ Y_{L2}^{G2})=\frac{Y_{L2}^1}{Y_{L2}^1-Y_{L2}^{G2}},\ m_3^1=(0\ Y_{L3}^1\ \infty\ Y_{L3}^{G3})=\frac{Y_{L3}^1}{Y_{L3}^1-Y_{L3}^{G3}}.$$

For points Y_{L1}^{G1}, Y_{L2}^{G2}, and Y_{L3}^{G3}, the coordinates $m_1=m_2=m_3=\infty$ define the sense of the plane of infinity $G_1G_2G_3$.

In addition to nonuniform coordinates m_1^1, m_2^1, m_3^1 of a point M^1, there are homogeneous projective coordinates $\xi_1^1, \xi_2^1, \xi_3^1, \xi_4^1$, which are set by a coordinate tetrahedron and a unit point SC. These homogeneous coordinates are defined as the ratio of the distances $\delta_1^1, \delta_2^1, \delta_3^1, \delta_4$ for the point M^1 and distances $\delta_1^{SC}, \delta_2^{SC}, \delta_3^{SC}, \delta_4^{SC}$ for a unit point SC to the planes of the coordinate tetrahedron $0G_1G_2G_3$.

Then, the distances δ_1^1, δ_1^{SC} correspond to the $0G_2G_3$ plane; therefore, $\delta_1^1=I_1^1$, $\delta_1^{SC}=I_1^{SC}$. Similarly, δ_2^1, δ_2^{SC} match the plane $0G_1G_3$ and $\delta_2^1=I_2^1$, $\delta_2^{SC}=I_2^{SC}$. Also, δ_3^1, δ_3^{SC} correspond to $0G_1G_2$ and $\delta_3^1=I_3^1$, $\delta_3^{SC}=I_3^{SC}$; δ_4^1, δ_4^{SC} agree with the plane $G_1G_2G_3$.

Therefore, we obtain

$$\rho\,\xi_1^1=\frac{\delta_1^1}{\delta_1^{SC}}=\frac{I_1^1}{I_1^{SC}},\quad \rho\,\xi_2^1=\frac{\delta_2^1}{\delta_2^{SC}}=\frac{I_2^1}{I_2^{SC}},\quad \rho\,\xi_3^1=\frac{\delta_3^1}{\delta_3^{SC}}=\frac{I_3^1}{I_3^{SC}},\quad \rho\,\xi_4^1=\frac{\delta_4^1}{\delta_4^{SC}},$$

where ρ is a coefficient of proportionality.

For finding the distances δ_4^1, δ_4^{SC}, the equation of $G_1G_2G_3$ is used

$$\frac{I_1^\infty}{I_1^{G1}}+\frac{I_2^\infty}{I_2^{G2}}+\frac{I_3^\infty}{I_3^{G3}}-1=0.$$

Then,

$$\mu_4\delta_4^1=\frac{I_1^1}{I_1^{G1}}+\frac{I_2^1}{I_2^{G2}}+\frac{I_3^1}{I_3^{G3}}-1,\qquad \mu_4\delta_4^{SC}=\frac{I_1^{SC}}{I_1^{G1}}+\frac{I_2^{SC}}{I_2^{G2}}+\frac{I_3^{SC}}{I_3^{G3}}-1,$$

where μ_4 is a normalizing factor.

Similarly to (5), (7), and (8), we immediately obtain

$$I_1^1 = \frac{I_1^{SC} m_1^1}{\frac{I_1^{SC}}{I_1^{G1}}(m_1^1 - 1) + \frac{I_2^{SC}}{I_2^{G2}}(m_2^1 - 1) + \frac{I_3^{SC}}{I_3^{G3}}(m_3^1 - 1) + 1},$$

$$I_2^1 = \frac{I_2^{SC} m_2^1}{\frac{I_1^{SC}}{I_1^{G1}}(m_1^1 - 1) + \frac{I_2^{SC}}{I_2^{G2}}(m_2^1 - 1) + \frac{I_3^{SC}}{I_3^{G3}}(m_3^1 - 1) + 1},$$

$$I_3^1 = \frac{I_3^{SC} m_3^1}{\frac{I_1^{SC}}{I_1^{G1}}(m_1^1 - 1) + \frac{I_2^{SC}}{I_2^{G2}}(m_2^1 - 1) + \frac{I_3^{SC}}{I_3^{G3}}(m_3^1 - 1) + 1}.$$

Let the subsequent regime correspond to point M^2 with loads $Y_{L1}^2, Y_{L2}^2, Y_{L3}^2$. The non-uniform coordinates

$$m_1^2 = \frac{Y_{L1}^2}{Y_{L1}^2 - Y_{L1}^{G1}}, \quad m_2^2 = \frac{Y_{L2}^2}{Y_{L2}^2 - Y_{L2}^{G2}}, \quad m_3^2 = \frac{Y_{L3}^2}{Y_{L3}^2 - Y_{L3}^{G3}}.$$

The regime change has the form

$$m_1^{21} = m_1^2 \div m_1^1, \qquad m_2^{21} = m_2^2 \div m_2^1, \qquad m_3^{21} = m_3^2 \div m_3^1.$$

Similarly to (9) we get the subsequent currents

$$I_1^2 = \frac{\rho\, I_1^2}{\rho\, 1} = \frac{I_1^1 \cdot m_1^{21}}{\frac{I_1^1}{I_1^{G1}} \cdot (m_1^{21} - 1) + \frac{I_2^1}{I_2^{G2}} \cdot (m_2^{21} - 1) + \frac{I_3^1}{I_3^{G3}} \cdot (m_3^{21} - 1) + 1},$$

$$I_2^2 = \frac{\rho\, I_2^2}{\rho\, 1} = \frac{I_2^1 \cdot m_2^{21}}{\frac{I_1^1}{I_1^{G1}} \cdot (m_1^{21} - 1) + \frac{I_2^1}{I_2^{G2}} \cdot (m_2^{21} - 1) + \frac{I_3^1}{I_3^{G3}} \cdot (m_3^{21} - 1) + 1},$$

$$I_3^2 = \frac{\rho\, I_3^2}{\rho\, 1} = \frac{I_3^1 \cdot m_3^{21}}{\frac{I_1^1}{I_1^{G1}} \cdot (m_1^{21} - 1) + \frac{I_2^1}{I_2^{G2}} \cdot (m_2^{21} - 1) + \frac{I_3^1}{I_3^{G3}} \cdot (m_3^{21} - 1) + 1}.$$

The obtained relationships are directly generalized to any number of loads and possess a group property.

4. General case of an active multi-port

Let us consider the general case of an active multi-port in Fig. 5. The circuit is also described by system of equations (10). Similarly to the above, we get the equations of three bunches of planes. Crossing of the planes of one bunch among themselves defines a bunch axis.

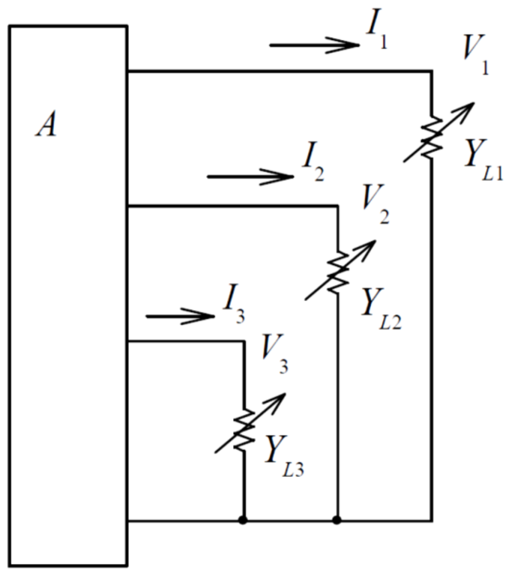

Fig. 5. General case of an active multi-port.

The equation of the axis of bunch Y_{L1} corresponds to the condition of $I_1 = 0$, $V_1 = 0$ and to equation $(I_2, I_3) = 0$. Therefore, this axis is located in the I_2, I_3 plane in Fig. 6a.

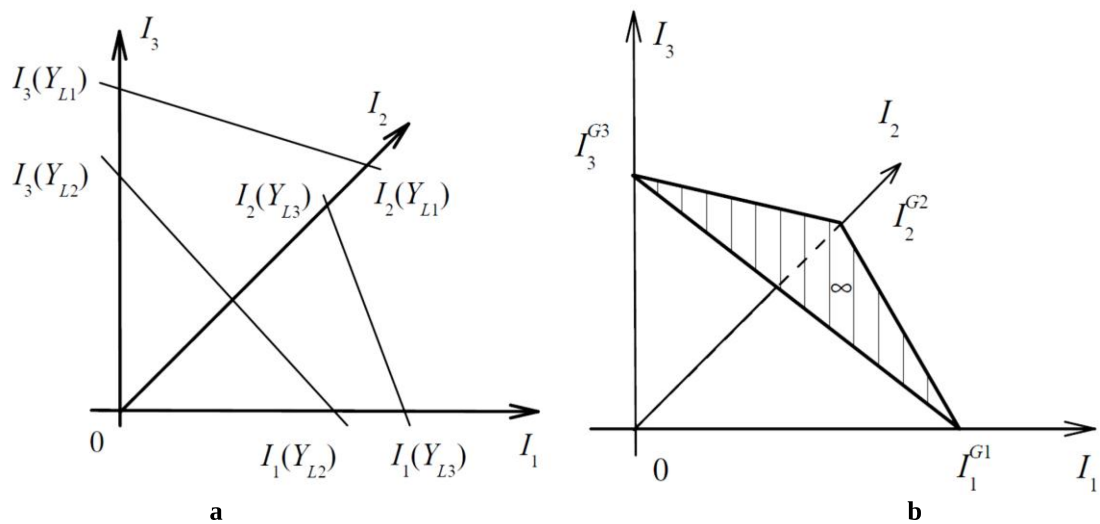

Fig. 6. Points of intersection do not coincide (**a**).
Points of intersection coincide and form the plane of infinity ∞ (**b**).

Points $I_2(Y_{L1})$, $I_3(Y_{L1})$ are the points of intersection with the respective axis. Similarly, we obtain points $I_1(Y_{L2})$, $I_3(Y_{L2})$ of intersection of bunch axis Y_{L2} and points $I_1(Y_{L3})$, $I_2(Y_{L3})$ of intersection of bunch axis Y_{L3}.

On the other hand, the projective system of coordinates must represent a tetrahedron $0, I_1^{G1}, I_2^{G2}, I_3^{G3}$ in Fig. 6b. Therefore, the following conditions must be satisfied:

$$I_1(Y_{L2}) = I_1(Y_{L3}) = I_1^{G1},\ I_2(Y_{L1}) = I_2(Y_{L3}) = I_2^{G2},\ I_3(Y_{L1}) = I_3(Y_{L2}) = I_3^{G3}.$$

We should determine requirements for Y -parameters. To this end, let us consider the base point or base values, $I_1 = I_1^{G1}$, $V_1 = V_1^{G1}$. Then, $I_2 = 0, V_2 = 0$, $I_3 = 0, V_3 = 0$.

Using (10), we get

$$\begin{cases} I_1^{G1} = -Y_{11}V_1^{G1} + I_1^{SC} \\ 0 = Y_{12}V_1^{G1} + I_2^{SC} \\ 0 = Y_{13}V_1^{G1} + I_3^{SC} \end{cases}.$$

From here, the requirements have the form

$$-V_1^{G1} = \frac{I_2^{SC}}{Y_{12}} = \frac{I_3^{SC}}{Y_{13}}.$$

Similarly, we consider that $I_2 = I_2^{G2}$, $V_2 = V_2^{G2}$.

Then

$$\begin{cases} 0 = Y_{12}V_2^{G2} + I_1^{SC} \\ I_2^{G2} = -Y_{22}V_2^{G2} + I_2^{SC} \\ 0 = Y_{23}V_2^{G2} + I_3^{SC} \end{cases}, \qquad -V_2^{G2} = \frac{I_1^{SC}}{Y_{12}} = \frac{I_3^{SC}}{Y_{23}};$$

We consider that $I_3 = I_3^{G3}$, $V_3 = V_3^{G3}$ and obtain

$$\begin{cases} 0 = Y_{13}V_3^{G3} + I_1^{SC} \\ 0 = Y_{23}V_3^{G3} + I_2^{SC} \\ I_3^{G3} = -Y_{33}V_3^{G3} + I_3^{SC} \end{cases}, \qquad -V_3^{G3} = \frac{I_1^{SC}}{Y_{13}} = \frac{I_2^{SC}}{Y_{23}}.$$

The obtained requirements or base points (2)–(5) are formally generalized to any number of loads.

We will identify this distributed network as a balanced network for output terminals.

The obtained conductivity values do not limit especially the functional possibility of these circuits, but allow essentially simplifying the calculation and recalculation of currents, as shown above.

Using the network in Fig. 3, we obtain an example of the general case of multi- port in Fig. 7.

If we choose the values of conductivities y_{12}, y_{13}, y_{23} by the required conditions, the balanced network is obtained.

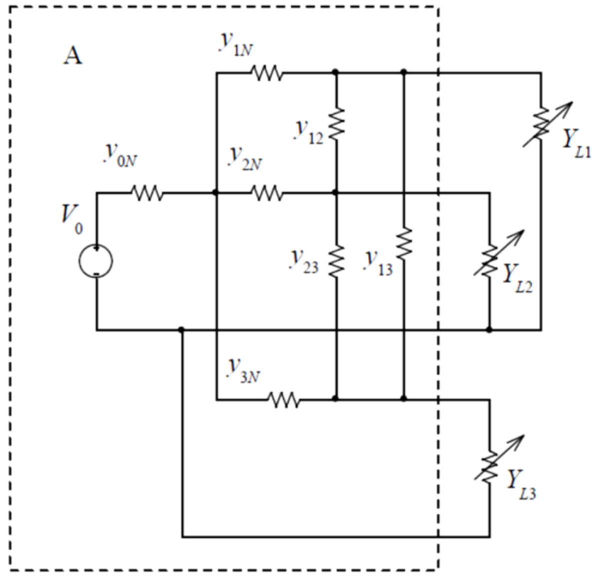

Fig. 7. Example of the general case of multi-port.

5. Conclusions

1. Active two-ports are always the balanced networks.
2. Active multi-ports with any number of loads can be balanced.
3. Application of projective coordinates allows obtaining convenience formulas for the recalculation of load currents.

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